

A practical parametrisation of line shapes of near-threshold resonances

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2016 J. Phys.: Conf. Ser. 675 022016

(<http://iopscience.iop.org/1742-6596/675/2/022016>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 134.94.122.86

This content was downloaded on 17/11/2016 at 12:23

Please note that [terms and conditions apply](#).

You may also be interested in:

[Near-threshold eta production in pp collisions](#)

Lü Qi-Fang and Li De-Min

[Near-threshold electron elastic scattering cross sections for Ta, W, Re, Mo, Tc and Rh atoms:](#)

[Determination of electron affinities and Ramsauer-Townsend minima](#)

Zineb Felfli, Alfred Msezane and Dmitri Sokolovski

[Effect of Near-Threshold Ionization on Electron Attachment in Gaseous Dielectrics](#)

Hirotake Sugawara, Takuya Ishigaki, Yuuki Hirochi et al.

[Geosynchrotron radio emission from CORSIKA-simulated air showers](#)

T Huege, R Ulrich and R Engel

[The second dissociation threshold boundlevels of hydrogen molecule](#)

Zhang Yan-Peng, Song

Jian-Ping, Gan Chen-Li et al.

[Differential cross sections for the electron-impact near-threshold electronic excitation of argon](#)

S Mondal, J Lower, S Buckman et al.

[Auger Electron Spectroscopy of Clean Polycrystalline Gold Films](#)

R. J. Reid

A practical parametrisation of line shapes of near-threshold resonances

C Hanhart¹, Yu S Kalashnikova², P Matuschek¹, R V Mizuk^{2,3},
A V Nefediev^{2,3} and Q Wang¹

¹Forschungszentrum Jülich, Institute for Advanced Simulation, Institut für Kernphysik (Theorie) and Jülich Center for Hadron Physics, D-52425 Jülich, Germany

²Institute for Theoretical and Experimental Physics, 117218, B.Chermushkinskaya 25, Moscow, Russia

³National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway 31, Moscow, 115409, Russia

E-mail: nefediev@itep.ru, q.wang@fz-juelich.de

Abstract. A practical parametrisation for the line shapes of near threshold resonance(s) is derived in the framework of a coupled-channel model which includes an arbitrary number of elastic and inelastic channels as well as a bare pole term. The parameters have a direct relation to phenomenology and can be employed to study the nature of the near-threshold states. The resulting analytical parametrisation is therefore ideally suited to investigate the full information content provided by the measurements and to establish a link between the experimental data and their theoretical interpretation.

1. Introduction

Since 2003 when Belle Collaboration announced the first observation of the meson-like state $X(3872)$ in the spectrum of charmonium [1], many other charmonium- and bottomonium-like states were observed which do not fit into the quark-model picture. At present there is no consensus in the literature regarding their nature.

Some of the new states reside in the vicinity of strong S -wave thresholds and they are seen in both open-flavour (elastic) and hidden-flavour (inelastic) final states. Since vast and detailed information is becoming available from the existing experiments and even more precise data are expected from future high statistics and high precision ones [2, 3, 4, 5], adequate theoretical parametrisations for the data analysis are urgently called for. It is important that such parametrisations are consistent with the requirements of unitarity and analyticity and allow one to exploit the full information content provided by the measurements. This is possible only via a simultaneous analysis of the data in all available channels.



2. Coupled-channel problem and a practical parametrisation for the line shapes

We present a coupled-channel approach based on the Lippmann-Schwinger equation (LSE) for the t -matrix, t . The potential is taken in a form consistent with phenomenology [6],

$$\hat{V} = \begin{array}{ccc} \text{Pole} & \beta=\overline{1, N_e} & i=\overline{1, N_{in}} \\ \begin{pmatrix} 0 & f_\beta & \lambda_i(k_i^{in})^{l_i} \\ f_\alpha & v_{\alpha\beta} & g_{i\alpha}(k_i^{in})^{l_i} \\ \lambda_j(k_j^{in})^{l_j} & g_{j\beta}(k_j^{in})^{l_j} & 0 \end{pmatrix} & & \end{array} \begin{array}{c} \text{Pole} \\ \alpha = \overline{1, N_e} \\ j = \overline{1, N_{in}} \end{array} . \quad (1)$$

It contains various types of interaction between the bare pole (labeled as “0”, for example, its position is M_0), the set of N_e elastic open-flavour channels $(Q\bar{q})(q\bar{Q})$ (here Q and q denote a heavy and a light quark, respectively) labeled by Greek letters, and a set of N_{in} inelastic hidden-flavour channels $(Q\bar{Q})(q\bar{q})$ referred to by Latin letters. The direct interactions in the inelastic channels are neglected, since they are expected to be weak.

The elastic potential $v_{\alpha\beta}$ is approximated by a constant matrix, and the elastic-inelastic transition form factors are taken in a separable form. Here k_i^{in} is the relative momentum in the i -th inelastic final state (the corresponding reduced mass and the threshold are denoted as μ_i^{in} and $m_{th_i^{in}}$, respectively). The omission of rescatterings within the inelastic channels allows us to disentangle the latter from the elastic channels and from the pole term. Thus we define

$$V_{00} = - \sum_i \lambda_i^2 J_i, \quad V_{\alpha 0} = f_\alpha - \sum_i g_{i\alpha} J_i \lambda_i, \quad V_{\alpha\beta} = v_{\alpha\beta} - \sum_i g_{i\alpha} J_i g_{i\beta},$$

where the real part of the inelastic loop integral

$$J_i = \int |\mathbf{q}|^{2l_i} S_i(\mathbf{q}) d^3q \rightarrow \frac{i(2\pi)^2}{\sqrt{s}} m_{th_i^{in}} \mu_i^{in} (k_i^{in})^{2l_i+1} \quad (2)$$

is omitted since it only renormalises parameters of the interaction. For convenience we define the effective potentials

$$V_{\alpha\beta}^{\text{eff}} = v_{\alpha\beta} - G_{\alpha\beta} - V_{\alpha 0} G_0 V_{0\beta}, \quad V_{\alpha 0}^{\text{eff}} = V_{\alpha 0} (1 + G_0 V_{00}), \quad G_{\alpha\beta} = \sum_i g_{i\alpha} J_i g_{i\beta},$$

where $G_0 = 1/(M_0 - M + V_{00} - i0)$ and arrive at a pair of decoupled LSE

$$\begin{aligned} t_{\alpha\beta} &= V_{\alpha\beta}^{\text{eff}} - \sum_\gamma V_{\alpha\gamma}^{\text{eff}} J_\gamma t_{\gamma\beta}, \quad t_{\alpha 0} = V_{\alpha 0}^{\text{eff}} - \sum_\beta V_{\alpha\beta}^{\text{eff}} J_\beta t_{\beta 0}, \\ J_\alpha &= \int S_\alpha(\mathbf{p}) d^3p = (2\pi)^2 \mu_\alpha (\kappa_\alpha + i k_\alpha) \equiv R_\alpha + i I_\alpha, \end{aligned} \quad (3)$$

with μ_α and k_α being the reduced mass and the relative momentum in the α 's elastic channel, respectively, $k_\alpha = \sqrt{2\mu_\alpha(M - m_{th_\alpha}) + i\epsilon}$, where m_{th_α} is the position of the α -th elastic threshold. With the help of the formalism of [7, 8] the solution of equation (3) can be written in the form

$$t_{\alpha\beta} = t_{\alpha\beta}^w + \frac{\phi_\alpha \bar{\phi}_\beta}{M - M_0 + \mathcal{G}_0}, \quad t_{\alpha 0} = \frac{M - M_0}{M - M_0 + \mathcal{G}_0} \phi_\alpha,$$

where

$$\begin{aligned} t^w &= t^v + \psi [\mathcal{G} - G^{-1}]^{-1} \bar{\psi} \\ \psi_{\alpha\beta} &= \delta_{\alpha\beta} \chi_\alpha - t_{\alpha\beta}^v J_\beta, \quad \bar{\psi}_{\alpha\beta} = \delta_{\alpha\beta} \chi_\alpha - J_\alpha t_{\alpha\beta}^v, \end{aligned}$$

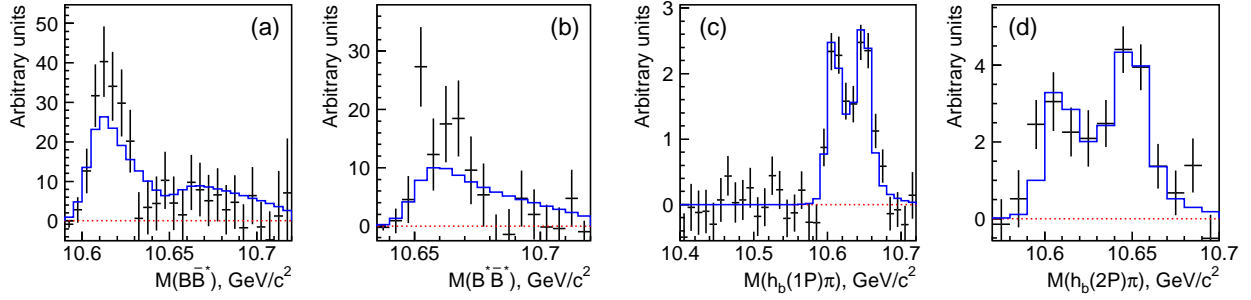


Figure 1. Fitted line shapes of the $Z_b(10610)$ and $Z_b(10650)$ in the $B^{(*)}\bar{B}^*$ and $\pi h_b(mP)$ ($m = 1, 2$) channels.

$$\begin{aligned}\mathcal{G}_{\alpha\beta} &= J_\alpha \psi_{\alpha\beta} = \bar{\psi}_{\alpha\beta} J_\beta = \delta_{\alpha\beta} J_\alpha - J_\alpha t_{\alpha\beta}^v J_\beta, \\ \phi_\alpha &= V_{\alpha 0} - \sum_\beta t_{\alpha\beta}^w J_\beta V_{\beta 0}, \quad \bar{\phi}_\alpha = V_{0\alpha} - \sum_\beta V_{0\beta} J_\beta t_{\beta\alpha}^w, \\ \mathcal{G}_0 &= \sum_i \lambda_i^2 J_i + \sum_\alpha V_{0\alpha} J_\alpha \phi_\alpha = \sum_i \lambda_i^2 J_i + \sum_\alpha \bar{\phi}_\alpha J_\alpha V_{\alpha 0}.\end{aligned}$$

Thus the entire problem is reduced to inverting a matrix as small as $N_e \times N_e$ — all other components of the t -matrix are algebraically related to $t_{\alpha\beta}$.

Since our knowledge of most resonance properties comes from production experiments, we build the production amplitude in the channel x as

$$\mathcal{M}_x = - \sum_\beta \begin{array}{c} \nearrow \\ \searrow \end{array} \begin{array}{c} p_1 \\ p_2 \end{array} \begin{array}{c} \nwarrow \\ \swarrow \end{array} \begin{array}{c} p_3 \end{array} t_{\beta x} = - \sum_\beta \mathcal{F}_\beta J_\beta t_{\beta x}, \quad (4)$$

where it was assumed that the production proceeds through the N_e point-like elastic sources \mathcal{F}_α and the Born term was neglected that is justified by the existence of near-threshold poles in the t -matrix. Neglecting the FSI with particle 3 and partially integrating over the phase space one can arrive at the formulae

$$\frac{d\text{Br}_\alpha^e}{dM} = \Lambda \left| \sum_\beta \xi_\beta t_{\beta\alpha} \right|^2 p_3 k_\alpha, \quad \frac{d\text{Br}_i^{\text{in}}}{dM} = \Lambda \left| \sum_\alpha \xi_\alpha t_{\alpha i} \right|^2 p_3 (k_i^{\text{in}})^{2l_i+1} \quad (5)$$

for the elastic and inelastic differential rates, where the allowed parameter range for M is given by $M_{\min} = m_1 + m_2$ and $M_{\max} = M_{\text{tot}} - m_3$, p_3 is the spectator momentum, and new parameters $\Lambda = \mathcal{F}_1^2$ and $\xi_\alpha = \mathcal{F}_\alpha / \mathcal{F}_1$ were introduced. Since for all elastic channels the range of forces is described by the same physics, it is natural to use $R_\alpha = (2\pi)^2 \mu_\alpha \kappa$.

3. Line shapes for the $Z_b(10610)$ and $Z_b(10650)$ states

As an application for our approach we consider the isovector 1^+ states $Z_b(10610)$ and $Z_b(10650)$ residing near the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, respectively, that are produced in $\Upsilon(5S)$ decays $\Upsilon(5S) \rightarrow \pi Z_b$ and are seen in seven decay channels: $Z_b \rightarrow B\bar{B}^*$, $B^*\bar{B}^*$, $\pi\Upsilon(nS)$ ($n = 1, 2, 3$), and $\pi h_b(mP)$ ($m = 1, 2$) [9, 10]. The quantum numbers of the final quarkonia fix the angular momenta of the inelastic final states to $l = 0$ for all $\pi\Upsilon(nS)$ final states and to $l = 1$ for the $\pi h_b(mP)$ final states. As an experimental input we use the background-subtracted and efficiency-corrected distributions in M for the $B^{(*)}\bar{B}^*$ and $\pi h_b(nP)$ channels [11, 9] with floating

normalisation in each channel, and the ratios of total branching fractions: $\text{Br}_{B\bar{B}^*}^e/\text{Br}_{B^*\bar{B}^*}^e$, $\text{Br}_i^{in}/\text{Br}_{B^*\bar{B}^*}^e$ [11, 9, 12, 13], where index i runs over all five inelastic channels $\pi\Upsilon(nS)$ and $\pi h_b(mP)$. We do not use the information on the $Z_b^{(\prime)}$ line shapes in the $\pi\Upsilon(nS)$ channels, since in the one-dimensional fit it is not possible to take into account correctly the interference with the nonresonant continuum, which is significant in the $\Upsilon(5S) \rightarrow \pi^+\pi^-\Upsilon(nS)$ transitions. To take into account the experimental resolution we convolve all the distributions with a Gaussian function with $\sigma = 6$ MeV.

The fact that the b -quark mass $m_b \gg \Lambda_{\text{QCD}}$ allows us to use heavy quark spin symmetry (HQSS) to reduce the number of parameters. In particular [14, 15],

$$\xi = \frac{g[\pi\Upsilon(5S)][B^*\bar{B}^*]}{g[\pi\Upsilon(5S)][B\bar{B}^*]} = -1, \quad \frac{g[\pi\Upsilon(nS)][B^*\bar{B}^*]}{g[\pi\Upsilon(nS)][B\bar{B}^*]} = -1, \quad \frac{g[\pi h_b(mP)][B^*\bar{B}^*]}{g[\pi h_b(mP)][B\bar{B}^*]} = 1. \quad (6)$$

In the same limit the direct interaction in the $B^{(*)}\bar{B}^*$ system can be parametrised in terms of two parameters γ_s and γ_t [6]. The bare pole is not included. Results of the simultaneous fit are shown in figure 1. Since the suggested parametrisation follows from the Lippmann-Schwinger approach for the coupled-channel problem then all unitarity and analyticity constraints for the t -matrix are fulfilled automatically. The inelastic channels enter additively that makes it particularly simple to extend the basis of the model and to include extra inelasticities. Finally, the developed parametrisation with the strict HQSS constraints imposed provides a good (CL=47%) description of the experimental data for the bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$.

4. Conclusions

We presented an approach which allowed us to build a practical parametrisation for the line shapes of near-threshold states consistent with all requirements of unitarity and analyticity. While additional effects such as finite widths of the constituents and the final state interaction with the spectator may also play a role, they can be included on top of the interactions considered in this work so that the presented parametrisation is expected to be realistic that is exemplified by the simultaneous fit to the data on the bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$.

Acknowledgments

R Mizuk and A Nefediev are supported by the Russian Science Foundation (Grant No. 15-12-30014).

References

- [1] Choi S K *et al.* (Belle Collaboration) 2003 *Phys. Rev. Lett.* **91** 262001
- [2] Abe T *et al.* 2010 (Belle-II Collaboration) *arXiv:1011.0352v1*
- [3] Drutskoy A G *et al.* *Eur. Phys. J. A* **49** 7
- [4] Asner D M *et al.* 2009 *Int. J. Mod. Phys. A* **24** S1
- [5] Lutz M F M *et al.* (PANDA Collaboration) 2009 *arXiv:0903.3905v1*
- [6] Hanhart C *et al.* 2015 *arXiv:1507.00382v1*
- [7] Baru V *et al.* 2010 *Eur. Phys. J. A* **44** 93
- [8] Hanhart C, Kalashnikova Yu S and Nefediev A V 2011 *Eur. Phys. J. A* **47** 101
- [9] Bondar A *et al.* (Belle Collaboration) 2012 *Phys. Rev. Lett.* **108** 122001
- [10] Adachi I *et al.* (Belle Collaboration) 2012 *arXiv:1209.6450v2*
- [11] Bondar A 2015 *Proc. XXVII Int. Symp. on Lepton Photon Interactions at High Energies (Ljubljana)* in press
- [12] Adachi I *et al.* (Belle Collaboration) 2012 *Phys. Rev. Lett.* **108** 032001
- [13] Garmash A *et al.* (Belle Collaboration) 2015 *Phys. Rev. D* **91** 7 072003
- [14] Bondar A E *et al.* 2011 *Phys. Rev. D* **84** 054010
- [15] Voloshin M B 2011 *Phys. Rev. D* **84** 031502